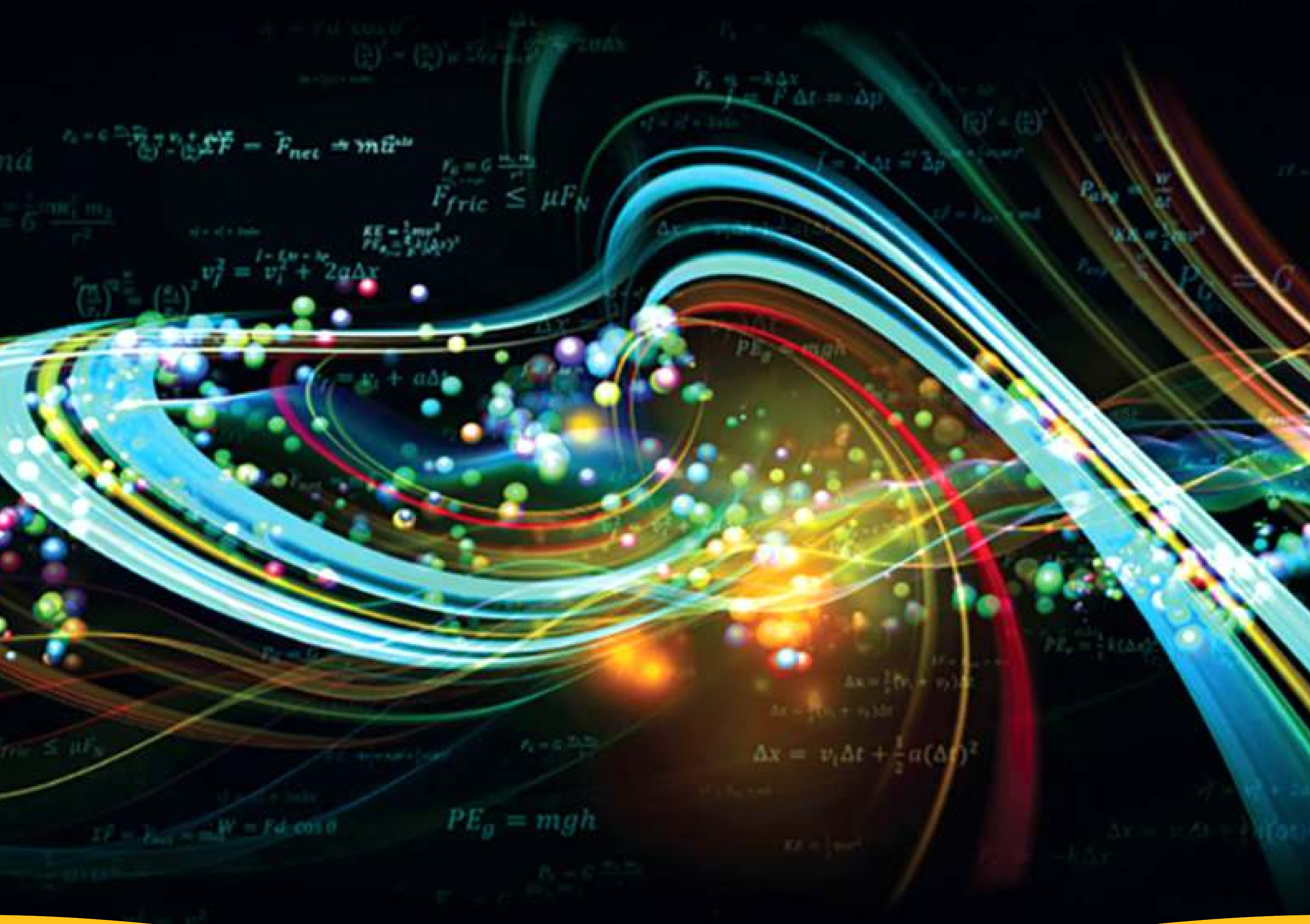
PHYSICS PEDAGOGY

PROBLEM SOLVING STRATEGIES USING WORKED EXAMPLES

VOLUME 3 - 2020

Compiled by Case Rijsdijk

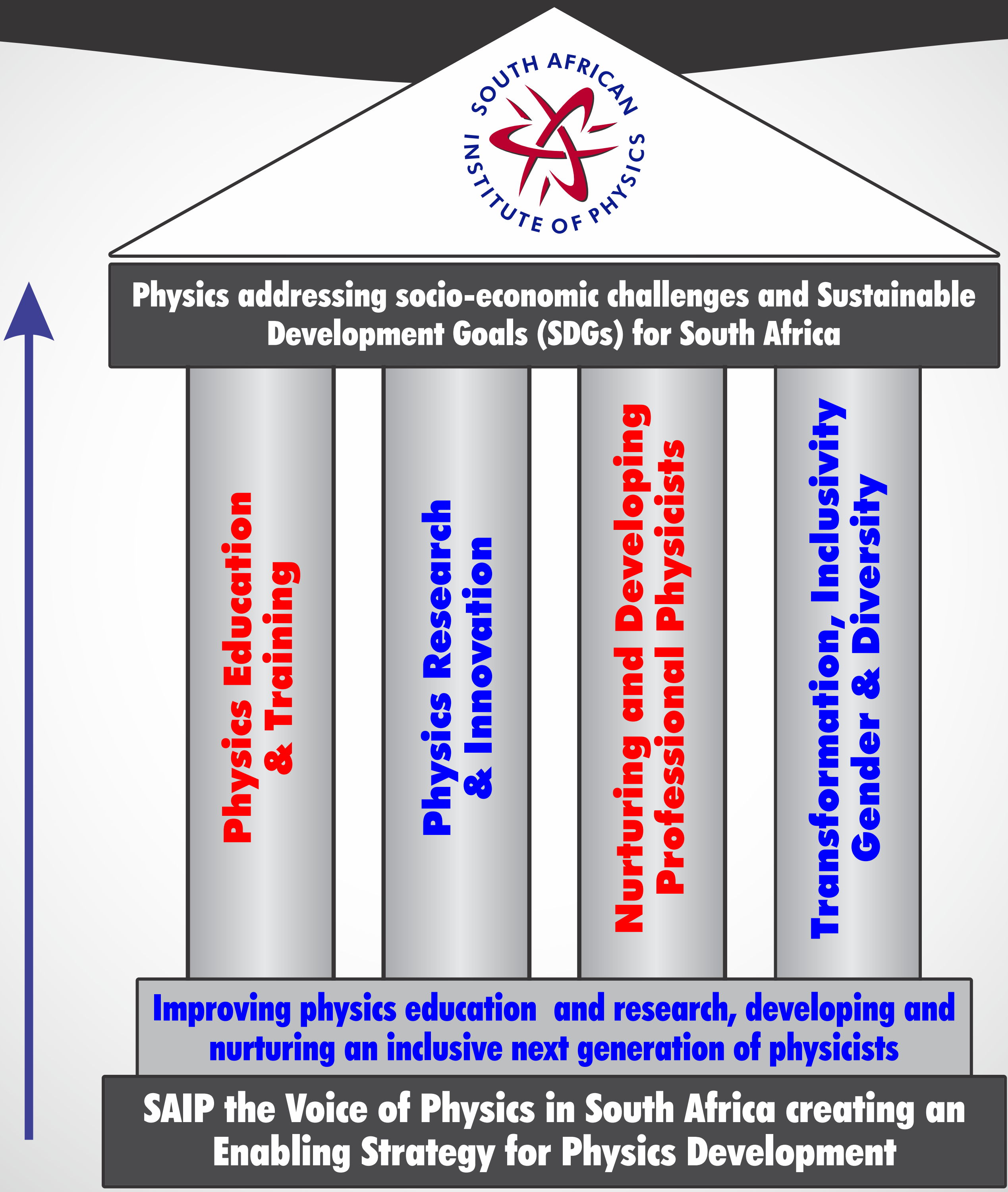


South African Institute of PHYSICS
THE VOICE OF PHYSICS IN SOUTH AFRICA

science & innovation
Department
Science and innovation
REPUBLIC OF SOUTH AFRICA

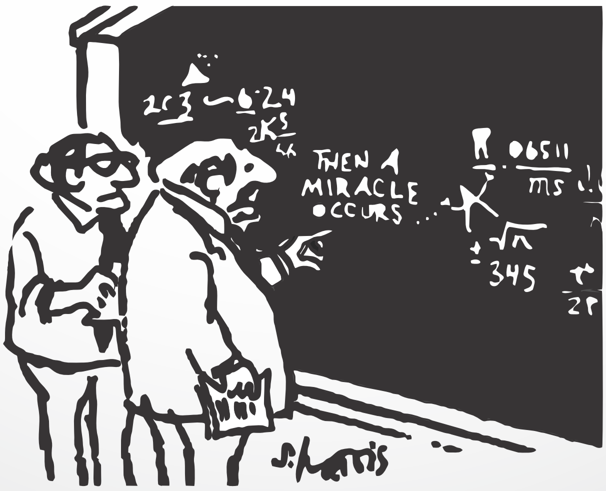
Philanthropy Initiative with Employees of Allan Gray

SAIP Value Proposition



Physics Pedagogy

Contents



"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO"

With thanks to Sidney Harris for putting humour into Physic

Physics Pedagogy

The original Physics Pedagogy was discussed as background in the first two volumes and will not be repeated here; for those interested see both Volumes 1 and 2 – which many will already have read and used!

1 Background

The primary aim with this series of booklets is to introduce a different approach to teaching students, and teachers, to come to terms with the concepts needed to understand Physics and solve Physics problems. It was found that using Multiple Choice Questions, MCQs, was a wonderful, interactive approach that encouraged group work and discussions.

This led to the two volumes of MCQs; Vol 1 covered Mechanics and Electricity and Vol 2 covering Matter, Waves, Sound, Light and General Physics – but neither contained any answers! This was deliberate; we all know that when one gets stuck with solving a problem, one looks at the answer and say *"Ah yes course"!* and then goes on without ever really understanding! The idea was for students and teachers to work together in teams to get the answer. If really stuck the tutor could be asked.

This volume goes a long way towards helping students and teachers, who for various reasons are working alone. Users should not try and look for the worked answer for the problem they are working on – the solution may not be here! I have made no effort in trying to "synchronize or match MCQ". With the worked examples in this volume – the selection is random. But the tools for problem solving are given often and repetitively, and in the process students and teachers learn how to tackle problem solving. Often there is more than one way to get the right answer, and many of these worked examples will show this.

2 Developing a Problem-Solving Strategy

Students need to develop a strategy to tackle problems and solve them, and it is difficult to do so without knowing more than one way to solve the problem. Most students have developed a habit of looking at a problem, decide what formula to apply, use the data given and produce an answer. What my Mathematics mentor, Prof S Skewes , called "Cookery Book Mathematics"; find the recipe, mix the ingredients, put it in the oven and out comes the cake!

Students generally don't use concepts to tackle a problem, primarily because they are unable to identify which ones to use or are relevant. They have a minimal grasp of concepts and are thus unable to use them to resolve the problem: they don't really understand the concepts or **how** to use them.

Below are some of the various types of knowledge that students need to know:

Conceptual knowledge, like the concept of momentum and energy, and how they differ in that both are concerned with velocity and mass. That the velocity of an object changes when it accelerates, or that the gravitational potential energy of an object decreases as it falls and gains kinetic energy.

Factual Knowledge, like the value of the gravitational constant G, the radius of the Moon, or the mass or velocity of an object.

Representational knowledge, like how to draw and use graphs.

Operational knowledge, like how to manipulate equations, resolve vectors and so on.

Procedural knowledge, for instance, if all forces are not involved with friction to then use the conservation of energy, or when finding potential energy, always specify a positive/negative, or when to apply Newton's Laws and draw a free-body diagram.

This last point is extremely important. Solving problems requires an understanding of what the problem really is!! The problem is most often set in words without the benefit of a diagram or illustration. So, the following approach is recommended:

2.1 Read the question carefully
2.2 Start drawing a diagram by interpreting the words and creating the diagram/sketch
2.3 Use supplementary things such as arrows, lines, angles etc
2.4 Put in the values that are given in the question.
2.5 Quite often this then describes the question diagrammatically – call this the **Situation** diagram.
2.6 In many cases a second, or even third, diagram may well need to be drawn in order to solve the problem, this could become the **Force**, **Vector** or **Free body diagram**

Summary

Diagram → Given Data → Basic Equation → Working Equations → Evaluation and Check

In solving problems, often the application of the Physics concepts will solve the problem, but to get the answer will require mathematics! See for example 2.8 below where the problem is discussed in detail

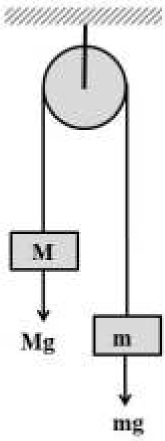
2.7 Example 1

A pulley is supported from a beam and two masses M and m (M > m) are connected by a thin rope passing over the pulley. Find the acceleration of the masses. Assume the pulley and string have no mass and the system is frictionless

Solution 1 – total mass approach. **Always** draw a diagram.

Difference in downward force **F** = M**g** – m**g** = (M + m)

So $a = \left(\dfrac{g\,(M - m)}{M + m}\right)$



Situation diagram

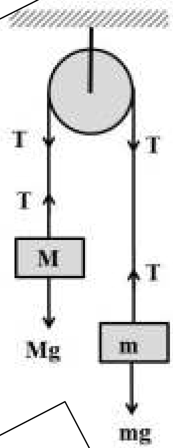
Solution 2

The free body approach. As usual draw a diagram, putting in all the forces acting on particle (or body) as shown. The tension **T** is the same throughout the string,

So $Mg - T = Ma$
and $T - mg = ma$

Adding these equations gives us $a = \left(\dfrac{g\,(M - m)}{M + m}\right)$. One might ask, what's the difference? Well if you were asked to find the tension T in the string, you would need to use one of the above equations.

Free body diagram



Force diagram

As the booklet is worked through there will be many situations where it might well be necessary to draw another diagram to fully expose the intricacies of the problem

As is the case here, one could argue "Why two diagrams"? This is a good question but teasing out the problem as much as possible should become a habit! As will be seen later, by putting everything into a single diagram often leads to getting the wrong answer!

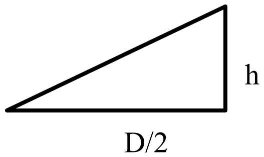
2.8 Example 2

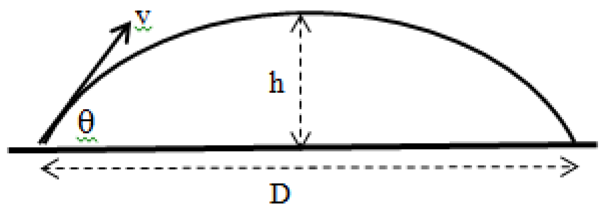
A ball is kicked on level ground at an angle θ. It lands after travelling D m and reaches a height h. The ratio h/D is:

A tan θ   B ½ tan θ   C ¼ tan θ   D 4 sin 2θ   E ¼ sin θ

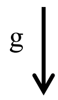
Solution C

Draw the diagram!! Clearly both h and D need to be found and solving this problem is **NOT** solving this triangle!





The Physics:



Find the vertical and horizontal components of the velocity v:

Vertical component $u_V = v\sin\theta$ Horizontal component $u_H = v\cos\theta$

To solve the problem, we need the time of flight, T, to determine h and D – then the Physics is done and the mathematics starts!

Now the time to greatest height, using the equation $V = U + at$ and noting g is positive down we get:

$0 = v\sin\theta - gt$ so $gt = v\sin\theta$ $t = \left(\frac{v\sin\theta}{g}\right)$ so the time of flight is $2t = T = \left(\frac{2v\sin\theta}{g}\right)$

To find h we *use* $V^2 = U^2 + 2ax$, substitute in the values to get:

$0 = v^2\sin^2\theta - 2gh$ so $h = \left(\frac{v^2\sin^2\theta}{2g}\right)$

And $D = v\cos\theta \times T$ or $D = v\cos\theta \times \left(\frac{2v\sin\theta}{g}\right)$

So $D = \left(\frac{2v^2\sin\theta\cos\theta}{g}\right)$ then the ratio $\left(\frac{h}{D}\right)$ looks quite hard!

But noting that dividing something by y, is the same as multiplying by $\left(\frac{1}{y}\right)$

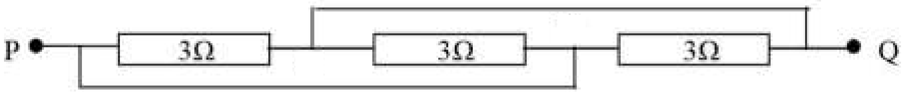
$$\text{So } \left(\frac{h}{D}\right) \text{ would be } \left(\frac{v^2\sin^2\theta}{2g}\right) \times \left(\frac{g}{2v^2\sin\theta\cos\theta}\right)$$

h	$\left(\frac{1}{D}\right)$

Then the v^2 cancel, as the does g and a $\sin\theta$ leaving $\left(\frac{h}{D}\right)=\frac{\tan\theta}{4}$ or ¼ $\tan\theta$

2.9 Example 3

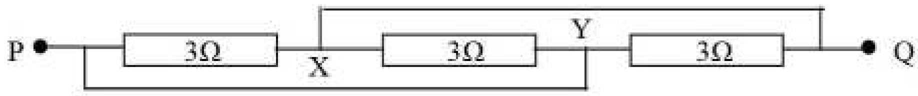
In the circuit shown below the resistance, in ohms (Ω), between the points P and Q is:



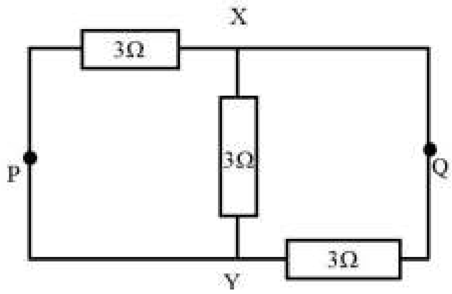
A 3/2 B 9 C 2/3 D 1

Solution **D**

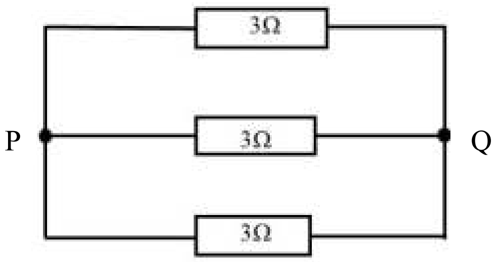
Many have a problem with interpreting this type of diagram correctly. Let's look at the same problem with a few extra labels:



Looking at the upper wire, the RHS has X connected to Q, which means that Q and X are the same point! The same applies to the lower wire, Y and P are the same point. So we could redraw the diagram as follows:

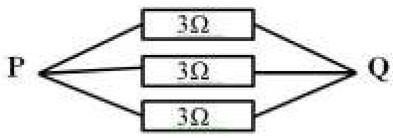


Which is another way of asking the same question, but it becomes easier to see the obvious. By simply sliding Y to P and X to Q you get:



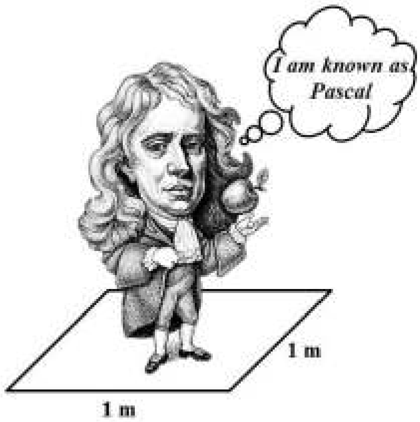
So the equivalent circuit is simply three, 3Ω resistors in parallel, so the equivalent resistance is 1Ω.

Students have got used to drawing circuits, as shown above, with straight lines at right angles to each other. But are many other ways of doing this, especially if a student is drawing a free hand sketch! Discuss the difference between these diagrams, especially the concept of a "line" being equivalent to a "point". Try drawing a few freehand sketches of different, but equivalent circuits.



3 General Properties of Matter

3.1 Problem



Pressure is defined as the force per unit area, or $N.m^{-2}$ where $1\ N.m^{-2} = 1$ Pascal (Pa).

Now the units of force are mass x acceleration which has dimensions $[M][L][T]^{-2}$ and area has dimensions of $[L]^2$, so a Pascal (or pressure) has dimensions of $[M][L]^{-1}[T]^{-2}$.

3.2 Problem

The equation of state for a real gas is given by $\left(P + \frac{a}{V^2}\right)(V - b) = RT$ where P, V and T are pressure, volume and temperature respectively and R is the universal gas constant. The dimensions of the constant *a* in the above equation are:

A $[M^{-1}L^{-5}T^{-2}]$ B $[ML^2T^{-2}]$ C $ML^5T^{-2}]$ D $[ML^3T^{-2}]$

Solution C

The term (a/V^2) has been added to the pressure, hence it should have the dimensions of pressure.

$$\left[\frac{a}{V^2}\right] = [P] \Rightarrow [a] = [V^2][P] = [L^6]\left[\frac{MLT^2}{L^2}\right] = [ML^5T^{-2}]$$

Obviously the dimensions of *b* are $[L]^3$

3.3 Problem

The Sun is half a degree (½ º) in diameter as seen from Earth, how long would it take to set as seen from a west facing beach at the equator? You can assume that the Sun sets vertically and that all optical effects can be neglected.

A 4 mins B 2 mins C 33.33 secs D 20 secs

Solution B

Sun moves 360º per day or 15º per hour = ¼ º per minute. So the Sun takes 2 minutes to set.

3.4 Problem

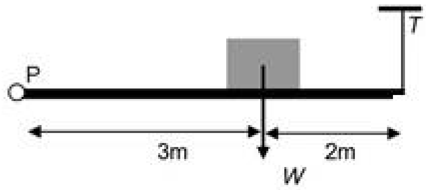
When a spring is stretched by 2 cm, its potential energy is U. If the spring is now stretched by 10 cm, its potential energy will be

A U/25 B U/5 C 5U D 25U

Solution D

The energy used is stretching a spring is $U = \frac{1}{2} kx^2$ where x is the extension and k is the spring constant. So if the extension is increases from 2 – 10 cm (5 times larger), the energy increases by $5^2 = 25$. Or you could say that the energy $U \propto x^2$

3.5 Problem



The diagram above shows a uniform beam of weight W pivoted at point P and supporting a block of weight W as shown. The tension T in the string is:

A $3W/5$ B $5W/2$ C $11W/6$ D $10W/11$ E $11W/10$

Solution **E**

Taking moments about P remembering that the plank weighs W that acts at 2.5 m

Clockwise 3W + 2.5W

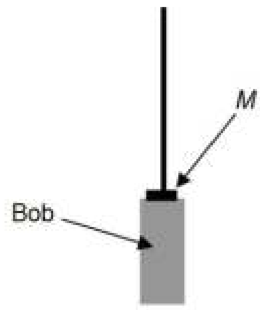
Anticlockwise 5T These are equal so 5T = 5.5W so $T = \frac{5.5W}{5} = 1.1W = \frac{11W}{10}$

You get the same answer if you use fractions.

Note. *Problems 3.6 and 3.12 use only conceptual knowledge and in finding the answer uses virtually no mathematics*

3.6 Problem

The diagram below shows a pendulum which is used to regulate a clock. The clock is running slow, so a physicist works out that if she were to place a ring with a pre-calculated mass M on the bob as shown, the clock would keep good time again. Which one of the following statements is true?



A It wouldn't work as the bob would then be heavier and the clock would slow even more.

B Adding the mass would make no difference since the period of a pendulum is independent of the mass of the bob.

C It wouldn't work as the only way to change the period of the pendulum is to change the length of the pendulum shaft.

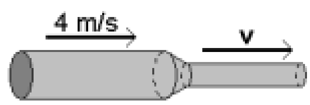
D By placing the ring on the bob, the effective length of the pendulum is shortened, and so decreases the period allowing the clock to keep proper time.

Solution **D**

The length of the pendulum is the length of the shaft plus the distance to the centre of mass of the bob. The centre of mass of both the bob and the ring are in their centre. So by placing the ring on top of the bob, effectively changes the position of the centre of mass upwards, thereby shortening the length of the pendulum and so decreasing the period so making the clock keep proper time.

3.7 Problem

Water flows from left to right through the pipe illustrated below. The cross-section of the pipe is circular. The diameter of the left half of the pipe is double the diameter in the right half of the pipe. If the speed of the water in the left half of the pipe is 4 $m.s^{-1}$, what is the speed of the water in the right half? (assume the liquid is incompressible, and 6+that there are no viscous or frictional forces)



A 2 $m.s^{-1}$ B 4 $m.s^{-1}$ C 8 $m.s^{-1}$ D 16 $m.s^{-1}$

Solution D

Assume the water flows through the pipe continuously. Water flows from the larger diameter to the smaller diameter pipe: from one of diameter D to one of diameter D/2. This means the area of the smaller pipe is ¼ the area of the larger pipe and therefore the volume of water that moves from the larger to the smaller pipe is the same, so its length should be 4X longer. This length of water must therefore move 4X faster, or 16 $m.s^{-1}$

3.8 Problem

The density of an object is carefully adjusted so that when it is placed halfway down in a tall cylinder of water at 4ºC it will remain where it is placed. When the temperature of the water changes the object will:

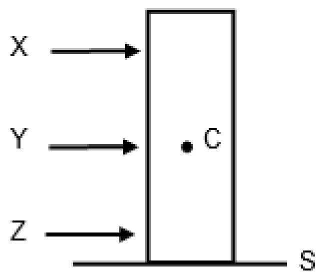
A rise B sink C stay where it is
D cannot say since you need to know whether the temperature is increased or decreased.

Solution B

The density of water is a maximum at 4º C. Therefore, **any** change of temperature would **decrease** the density of water, so the object would sink.

3.9 Problem

A solid metal cylinder stands on a smooth (frictionless) surface. It has a mass M and its centre of mass is at C. Which force, X, Y or Z, each larger than Mg, will cause the cylinder to topple over?



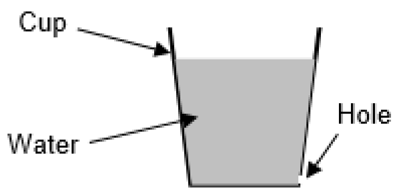
A X B Y C Z
D None of the forces will topple the cylinder.

Solution D

Since the cylinder is standing on a frictionless surface, it does not matter where a force is applied: the cylinder will always slide and not topple. Discuss what happens if the force is an impactive one?

3.10 Problem

A plastic cup is filled with water as shown below. The cup is allowed to fall freely. In which direction will the water come out of the hole as it falls?



A 

B 

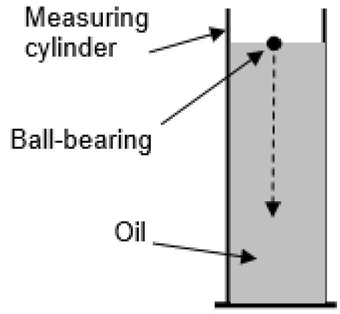
C 

D No water will come out of the hole.

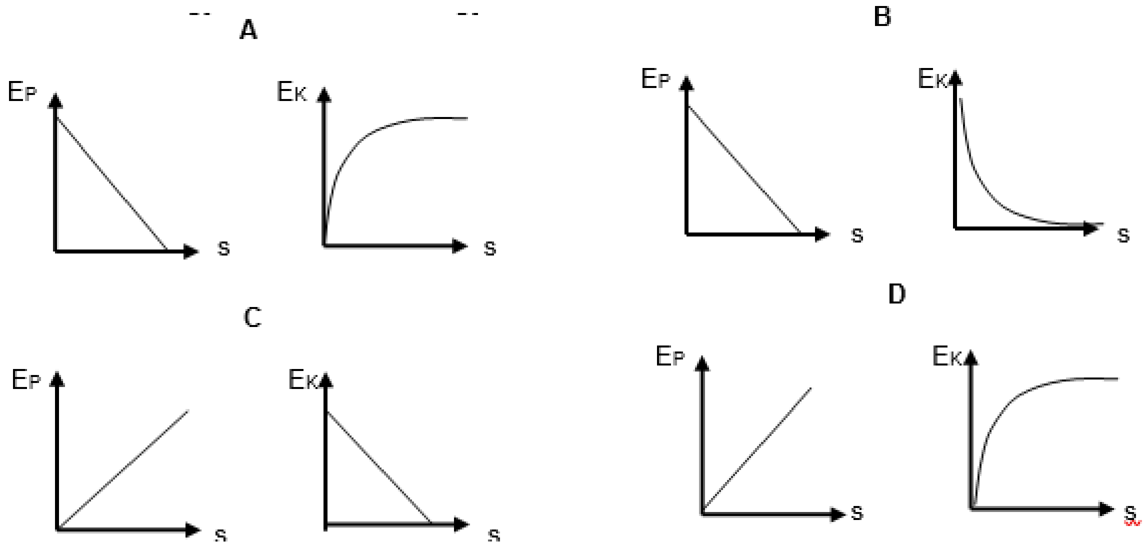
Solution D

A freely falling object accelerates at g, and is therefore effectively "weightless". So no water will fall out of the cup as there is no pressure gradient. Another way of looking at this is to say that the hole is falling as fast as the water trying to get out.

3.11 Problem



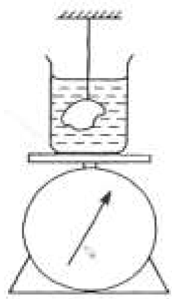
The diagram right shows a tall measuring cylinder filled with cooking oil. A small, steel ball-bearing is held on the surface of the oil and then released. Which one of the following pairs of Energy-Distance graphs correctly shows the change of potential and kinetic energy, E_P and E_K respectively?



Solution **A**

As the ball-bearing falls in the oil, its speed will increase in such a way that the downward acceleration will decrease and eventually be zero, and it will continue at a constant speed, known as its "terminal velocity" until it reaches the bottom of the cylinder. This means that the kinetic energy will increase to a steady maximum. The potential energy will simply decrease linearly with distance.

3.12 Problem



When a beaker of water rests on a balance, the weight indicated is X. A solid object of weight Y in air displaces weight Z of water when completely immersed What will be the balance reading when the object is suspended in the beaker of water so that it is totally immersed (but does not touch the beaker) as shown

A X   B X+Z   C X+Y-Z   D X+Y

Solution **B**

When an object is submerged in water it experiences an *up-thrust* equal to the mass (or weight) of the water displaced by that object. According to Newton's third law this *up-thrust* will have an equal *down-thrust* increasing the scale reading by the weight of water displaced. Therefore the scale will show X + Z.

4 Waves, Sound and Light

4.1 Problem

The intensity of a sound wave gets reduced to 80% of its original intensity on passing through a slab. The intensity of the sound wave on passing through two consecutive slabs is reduced to . . .

A 87% B 75% C 64% D 50%

. . . of the original intensity.

Solution C

If the intensity decreases by 80% then I' = 4/5I. This means that I" = 4/5 x 4/5 = 16/25. This is equivalent to 64%

4.2 Problem

The siren of an ambulance has a frequency of 700 Hz. You are standing on the pavement as the ambulance drives past you at a speed of 20 m.s^{-1}. The frequency that you will hear when the ambulance is approaching you is about: (Assume the speed of sound = 340 m.s^{-1})

A 661 Hz B 741 Hz C 744 Hz D 788 Hz

Solution B

Using a simplified Doppler equation $f_L = \left(\frac{c \pm v_S}{c \quad v_L}\right) f_S$ with $v_L = 0$

$$f_L = f_S\left(\frac{c + v_S}{c - v_L}\right) = 700\left(\frac{360}{340}\right) = \frac{700 \times 360}{340} = 741.17\text{Hz} \approx 741\,\text{Hz}$$

4.3 Problem

A parallel beam of light shines on a converging lens of focal length 200 mm. How far behind this lens would a second converging lens, of 150 mm focal length have to be placed so that the light would emerge from it as a parallel beam?

A 350 mm B 200 m C 150 mm D 50 mm

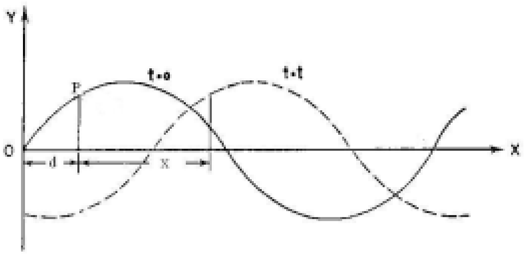
Solution A

First lens forms an image at the focal point, ie 200 mm from the lens. This image forms the "object" for the second lens, and if placed at the focal point of this lens, the emerging beam will be parallel.

So distance between lenses is 200 + 150 = 350 mm.

4.4 Problem

A wave has the equation, $y = 4 \sin (3\pi t - 6\pi x)$



This wave is travelling in the

A -x direction

B +x direction

C Cannot be answered unless the amplitude is given,

D No direction, it is a standing wave

Solution A

The general wave equation can be given by:

$$y = A \sin 2\pi vt$$

Where A = amplitude, v is the velocity at time t = 0. Sometime later, say t, the wave will have moved as shown by the broken line and the point P will have moved a distance of x. If the broken curve is now moved back a distance of – x it will coincide with the curve at time t = 0 and this can be written as:

$$y = A \sin 2\pi (vt - x)$$

This can be re-written as used in the question, since v, π etc are constants.:

$$y = 4 \sin (3\pi t - 6\pi x)$$

and represents a wave moving in the positive x-direction.

4.5 Problem

A metal rod AB, 2m long, is clamped ¼ m from B at a point O. It is then gently struck with a wooden mallet and produces a note of frequency 2 500Hz. The linear speed of the wave in the rod, is in $m.s^{-1}$:

A 1 250 B 2 500 C 5 000 D 10 000

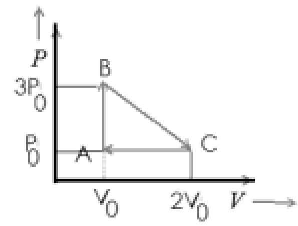
Solution C

The point O must be a node which means that the length OB is a ¼ wavelength, $\lambda/4$ and so the wavelength $\lambda = 2m$. Then using the wave equation:

$v = f\lambda$ we get that $v = 2500 \times 2 = 5000\ m.s^{-1}$

This problem comes in a variety of forms, often using guitar or violin strings, when the player will just touch a string to raise the pitch of the note. Also often used when talking of standing wave

5 Heat and Thermodynamics



5.1 Problem

According to the figure if one mole of an ideal gas is taken in a cyclic process, the work done by the gas in the process will be:

A P_0V_0 B $2P_0V_0$ C $3P_0V_0$ D $4P_0V_0$

Solution

Work done = area of a PV curve ie $w = \frac{1}{2}(3P_0 - P_0)(2V_0 - V_0) = P_0V_0$

5.2 Problem – extra-curricular

Calculate the change in entropy for 200 kg of water slowly heated from 20ºC to 80ºC.

A 1.16×10^6 J.K^{-1} B 1.56×10^5 J.K^{-1}
C 8.41×10^5 J. K^{-1} D 2.32×10^6 J. K^{-1}

Solution

Using* $\Delta S = m \times c \times \ln\left(\frac{T_2}{T_1}\right)$, you get $\Delta S = 200\,kg \times 4.2\ kJ.kg^{-1} \times \ln\left(\frac{353K}{293K}\right)$

So this becomes $\Delta S = 2 \times 10^5 \times 4.2 \times \ln 1.20 = 1.56 \times 10^5\ J.K^{-1}$

* From $S = \frac{Q}{T}$ so then $\Delta S = \int_i^f dS = \int_i^f \frac{dQ}{T}$ so $\Delta S = Q \cdot \ln\left(\frac{T_f}{T_i}\right) = m \times c \times \ln\left(\frac{T_f}{T_i}\right)$

A common error here is not to use absolute temperature; using Cº gives you answer A!

5.3 Problem

Some hot water was added to twice its mass of cold water at 10º C. The resulting temperature of the mixture was 20º C. What was the temperature of the hot water?

A 20º C B 30 º C C 40º C D 50º C

Solution C

Heat lost = heat gained.

Hot water loses heat; $mc\delta\theta = mc(\theta - 20)$
Cold water gains heat; $mc\delta\theta = 2mc(20 - 10)$

So $mc(\theta - 20) = 2mc(20 - 10)$ ie. $\theta - 20 = 2 \times 10$. So $\theta = 20 + 20 = 40$º C

5.4 Problem

How many joules of energy are required to heat a 200 g piece of copper from 20º C to 50º C? Assume the SHC of copper = 0.4KJ/Kg/K

A 2 240 000J B 2 400 C 240J D 9.6J

Solution B

From the data sheet we see that $Q = cm\Delta T$, where

c = the Specific Heap Capacity, SHC, (for Copper this is 400 $J.kg^{-1}\ K^{-1}$: this would normally be given
m = the mass = 200 g = 0.2 kg
ΔT = change in temperature = 30 K (322K – 293K) (be careful with units – make sure you use the correct ones)

So Q = 400 x 0.2 x 30 = 2 400 J

5.5 Problem

The temperature of an ideal gas is increased from 120 K to 480 K. If the rms velocity of the gas molecules at 120 K is v, then at 480 K this becomes:

A 4v B 2v C v/2 D v/4

Solution B

Using the gas equation $v_{RMS} = \sqrt{\left(\frac{3RT}{M}\right)}$, then at two temperatures T_1 and T_2 the rms velocities v_1 and v_2 are related by:

$$\frac{v_1}{v_2} = \sqrt{\frac{\left(3RT_1/M\right)}{\left(3RT_2/M\right)}} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{120}{480}} = \frac{1}{2}$$

$$2v_1 = v_2 = 2v$$

6 Vectors, Motion, Forces and Mechanics

6.1 Problem

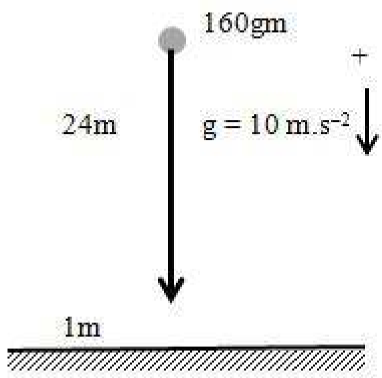
A cricket ball of mass 160 gm is dropped from the top of a building 25 m high and caught 1 m above the ground. What is the speed of the ball as the catcher catches it? You can ignore air resistance and take g = 10 $m.s^{-2}$

A 22.4 $m.s^{-1}$ B 21.7 $m.s^{-1}$ C 21.9 $m.s^{-1}$ D 22.1 $m.s^{-1}$

Solution 1 C

Using the equations of motion and assuming that the mass of the cricket ball has no effect – not proven – but generally accepted – should it be?

Note a diagram has been drawn and all the information needed has been added, including that down is +ve!



Since u = 0, v = v, a = g and s = 25 – 1 = 24 m. Then using:

$v^2 = u^2 + 2gx$ gives $v^2 = 0 + 2 \times 10 \times 24 = 480$.

So $v = \sqrt{480} = 21.9$ $m.s^{-1}$

Solution 2 C

Using the conservation of energy. This can be used as there are no energy losses due to friction (air resistance).

Now E_P = mgh and this will equal the kinetic energy as the ball is caught. Note that the ball only travels 24 m. Then taking down as +ve

$E_P = E_K$. So $mgh = \frac{1}{2} mv^2$ or $v^2 = 2gh$ so $v = \sqrt{(2 \times 10 \times 24)} = 21.9$ $m.s^{-1}$

Note: The speed of the ball does not depend on its mass – which, as legend has it, is what Galileo demonstrated when he allegedly dropped two different size cannon balls from the Leaning Tower of Pisa.

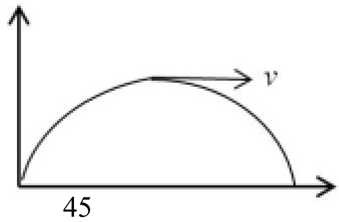
One could have assumed that fact and then simply used the equations of motion to find the speed of the ball without using the Conservation of Energy, but as will be seen later, there are many occasions that show that using the Conservation of Energy, leads to a faster and often easier solution, as the following two problems show.

6.2 Problem

A ball whose kinetic energy is E is thrown at an angle of 45° with the horizontal. What is its kinetic energy at the highest point of its flight? Ignore the effects of air resistance.

A 2E B $\sqrt{2}E$ C $\frac{E}{\sqrt{2}}$ D $\frac{E}{2}$

Solution 1 **D**



Using the conservation of energy as there is no air resistance. Note that again a simple diagram is drawn. See below for comments.

Initial kinetic energy = $\frac{1}{2}mu^2$

The horizontal speed/velocity is always $v = u\cos 45 = \frac{u}{\sqrt{2}}$, which will be the velocity at its highest point as well.

So the Kinetic Energy at the top is $\frac{1}{2}mv^2 = \frac{1}{2}m\left[\frac{u^2}{2}\right] = \frac{1}{2}\left[\frac{1}{2}mu^2\right] = \frac{E}{2}$

Solution 2 **D**

The initial velocity can be resolved into two components: vertical and horizontal. In this case some additional arrows indicating the directions of the vertical and horizontal components could be added at the origin.

Initial vertical component = u sin 45 $= \frac{u}{\sqrt{2}}$

Initial horizontal component is $v = u\cos 45 = \frac{u}{\sqrt{2}}$, and it should be remembered that this component has no acceleration, and as there is no air resistance, is therefore constant.

At its highest point the vertical component = 0, so only the speed at this point that needs to be considered to calculate E_K is the horizontal component!

So the energy at its highest point is $\frac{1}{2}mv^2 = \frac{1}{2}m\left[\frac{1}{\sqrt{2}}u\right]^2 = \frac{1}{2}\left[\frac{1}{2}mu^2\right] = \frac{E}{2}$ as before.

The vertical E_K energy lost:

Taking $V^2 = U^2 + 2as$ gives $0 = U^2 - 2gh$ and so E lost $mU^2 = 2mgh$ or ½ $mU^2 = mgh$
where $U = u/\sqrt{2}$ and so $U^2 = u^2/2$ so ½ $u^2/2$
Then if you were to drop the object from height h, it would gain E_K = ½ $mu^2/2$ ie E/2!!
By projecting it vertically you would lose E/2

There is a symmetry here as the angle of projection is 45o, so vertical and horizontal velocities are the same, so if you lose the vertical component of the energy, by symmetry you would simply lose half the energy – but only because air resistance = 0!!

6.3 Problem

Two balls are projected vertically upwards from the same point, the second 2 seconds after the first, with the same initial velocity of 40 $m \cdot s^{-1}$. The balls will collide at a height of:

A 200 m B 80 m C 75 m D 40 m

Solution C

Let the two balls collide t seconds after the first ball is thrown, and let h be the height at which they collide.

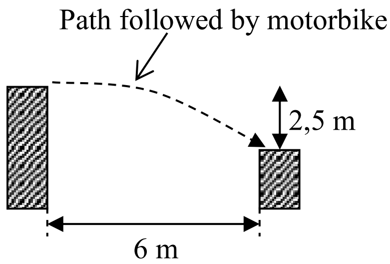
For the first ball: $h = 40 \times t - \frac{1}{2} gt^2$ For the second ball: $h = 40 \times (t - 2) - 5(t - 2)^2$

Putting h = h we get $40 \times t - 5t^2 = 40 \times (t - 2) - 5(t - 2)^2$

So 20t = 80 + 20 = 100, so t = 5 seconds; which gives $h = 40 \times 5 - 5 \times 5^2 = 75$ m

6.4 Problem

A daredevil on a motorbike jumps a river 6 m wide. He lands on the edge of the far bank, which is 2.5 m lower than the bank from which he takes off.. His minimum horizontal speed, in $m \cdot s^{-1}$, at takeoff must be



A 7.0 B 8.5 C 10.8 D 12.0

Solution B

The time it takes to fall 2.5m: $h = \frac{1}{2} at^2$ so $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{5}{10}} = \sqrt{0.5} = 0.71$s

Whilst in the air there is no horizontal acceleration, so the time speed needed to cover the 6 m of river is: $\frac{6}{0.71} = 8.5\ m \cdot s^{-1}$, rounded off to 2 significant figures.

Discuss some aspects of this problem within your group.

6.5 Problem

Using the principle of conservation of mechanical energy find the velocity with which a body must be projected, vertically upwards, from the Earth's surface, to reach a height of *R* above the Earth's surface. Assume the Earth's mass is *M* and its radius *R*. Neglect air resistance.

A $v = \sqrt{\frac{GM}{R}}$ B $v = \sqrt{\frac{2GM}{R}}$ C $v = 2GMR$ D $v = \left(\frac{GM}{2R}\right)^2$

Solution B

Using the principle of conservation of energy,

$mgh = \frac{1}{2} mv^2$ or using giving values $MgR = \frac{1}{2} Mv^2$ so $v = \sqrt{2gR}$

But $F = mg = \frac{GMm}{R^2}$ so $g = \frac{GM}{R^2}$ then from the above $v = \sqrt{\frac{2MG}{R}}$

6.6 Problem

Two projectiles are launched simultaneously with the same initial speed, in the same direction and from the same point on a horizontal plane. One is launched at 70° to the horizontal, and the other at 20° to the horizontal. Which one of the following statements is true, if the air resistance is neglected?

A The projectiles will land at the same time
B Both land at the same point.
C The projectile launched at 70° to the horizontal will land first.
D The horizontal component of each is constant and has the same value.

Solution 1 B

For the projectile launched at 70°

Vert component = vgsin70 = 0.94vg so time to reach max height using v = u + at is; $t = \frac{0.94v}{g}$ the time of flight is $2t = \frac{1.88v}{g}$ secs.

Horizontal comp = vcos70 = 0.34v

This means that the range is 0.34v x 2t ie. $0.34v \times 2t = \frac{0.34v \times 1.88v}{g} = \frac{0.64v^2}{g}$

For the projectile launched at 20°

Vert component the time of flight is $2t = \frac{0.64v}{g}$ secs.

Horizontal component is vcos 20 = 0.94v

This means that the range is 0.94v x 2t ie. $0.94v \times 2t = \frac{0.94v \times 0.68v}{g} = \frac{0.64v^2}{g}$

The range in both cases is the same so the projectiles land at the same place.

Solution 2 B

It also possible to do this making the assumption that the initial speed is, say, 20 $m \cdot s^{-1}$ and then solve the problem numerically

6.7 Problem

A car covers half distance between two points at 20 km/h and the remaining next half at 30 km/h. The average speed of the car is:

A 30 km/h B 28 km/h C 25 km/h D 24 km/h E 22 km/h

Solution D

Let the total distance be 2d, then the average speed V is defined by

$$V = \left(\frac{\text{Total Distance}}{\text{Total Time}}\right) = \left(\frac{D}{T}\right)$$

Now the average speed for the first half $v_1 = \left(\frac{d}{t_1}\right)$, and for the second half $v_2 = \left(\frac{d}{t_2}\right)$

This then means that $t_1 = \left(\frac{d}{v_1}\right)$ and $t_2 = \left(\frac{d}{v_2}\right)$.

$$D = 2d \text{ and } T = t_1 + t_2 = \frac{d}{v_1} + \frac{d}{v_2} = \left(\frac{dv_2 + dv_1}{v_1 v_2}\right) = d\left(\frac{v_2 + v_1}{v_1 v_2}\right)$$

$$V = \left(\frac{D}{T}\right) = \left(\frac{2d}{d\left(\frac{v_2 + v_1}{v_1 v_2}\right)}\right) = \left(\frac{2v_1 v_2}{v_1 + v_2}\right) \text{ or simply } V = \left(\frac{D}{T}\right) = \left(\frac{2v_1 v_2}{v_1 + v_2}\right) \ldots\ldots\ldots (1)$$

Then substituting in the values for v_1 and v_2 or 20 and 30 we get

$$V = \left(\frac{D}{T}\right) = \left(\frac{2 \times 20 \times 30}{20 + 30}\right) = \left(\frac{2 \times 600}{50}\right) = 2 \text{ x } 12 = 24 \text{ km/h}$$

Notes

1 Different values for the average velocity can be chosen to "change" the question
2 See also problems 6.14 and 9.5

6.8 Problem

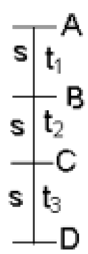
A, B, C and D are four points on the same vertical line such that AB = BC = CD. If a particle falls freely from rest at A, the time taken by it to describe AB, BC and CD are in the ratio of:

A 1:3:5 B 1:4:6 C 1:4:9 D 1:(√2 – 1):(√3 - √2)

Solution D

For motion from A to D,

$$3s = \tfrac{1}{2} g (t_1 + t_2 + t_3)^2 \Rightarrow t_1 + t_2 + t_3 = \sqrt{\frac{6s}{g}}$$



Similarly, for motion from A to C, $t_1 + t_2 = \sqrt{\frac{4s}{g}}$

For motion from A to B: $t_1 = \sqrt{\frac{2s}{g}}$

Then, $t_2 = \sqrt{\frac{2s}{g}}\left(\sqrt{2} - 1\right)$ and $t_3 = \sqrt{\frac{2s}{g}}\left(\sqrt{3} - \sqrt{2}\right)$

So $t_1 : t_2 : t_3 = 1 : \left(\sqrt{2} - 1\right) : \left(\sqrt{3} - \sqrt{2}\right)$

6.9 Problem

A stone is dropped into a well in which the water level is h m below the top. If the speed of sound is c and the acceleration due to gravity is g, then the time to hear the splash of the stone hitting the water is:

A $h\left[\sqrt{\frac{2}{gh}+\frac{1}{c}}\right]$ B $h\left[\sqrt{\frac{2}{gh}-\frac{1}{c}}\right]$

C $h\left[\frac{2}{g}+\frac{1}{c}\right]$ D $h\left[\frac{2}{g}-\frac{1}{c}\right]$

Solution A

Time of fall is given by $\sqrt{\frac{2h}{g}}$ .Time taken by the sound to return = $\frac{h}{c}$

This is effectively the end of the problem – the Physics is done!! To get the required answer a little mathematical manipulation needs to be done; this is often the case to make the answers less "obvious"!

So total time is $T = \sqrt{\frac{2h}{g}} + \frac{h}{c} = h\left[\sqrt{\frac{2}{gh}} + \frac{1}{c}\right]$

6.10 Problem

Two bodies of mass M and 4M are moving in a straight line, each with kinetic energy E. The ratio of their momenta is:

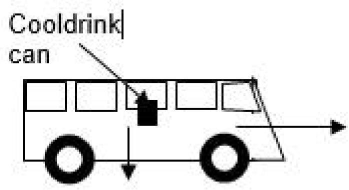
A 4:1 B 1:4 C 1:8 D 1:2

Solution D

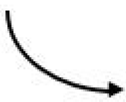
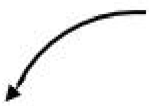
$$E_1 = \frac{1}{2}mv^2 = \frac{p_1^2}{2m} \text{ so } p_1 = \sqrt{2m_1E_1} \text{ and similarly, } E_2 = \frac{1}{2}mv^2 = \frac{p_2^2}{2m} \text{ so } p_2 = \sqrt{2m_2E_2}$$

Then the ratio $\frac{p_1}{p_2} = \frac{\sqrt{2m_1E_1}}{\sqrt{2m_2E_2}} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{m}{4m}} = \frac{1}{2}$

6.11 Problem



Thebe, travelling in a minibus in the direction shown, accidentally drops a cooldrink can out of the window as shown above. What is the correct path that the can takes in falling to the ground?

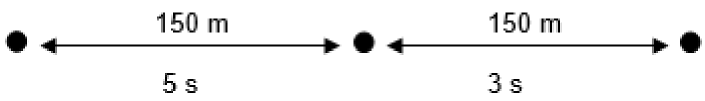
A  B  C  D 

Solution B

The instant the can is dropped it has a forward velocity equal to that of the minibus and the downward velocity is 0. In time the downward velocity increases (gravitational effect) and the horizontal velocity decreases (air resistance). This is an excellent question for group discussion!

6.12 Problem

A car accelerates along a straight road at a uniform rate. Next to the road are telephone poles 150 m apart as shown below. If the car covers the consecutive distances between three poles in 5 and 3 seconds respectively, then the acceleration of the car is, (in $m.s^{-2}$):



A 10.67 B 8 C 5.33 D 5

Solution D

Average speed occurs at half time. The average speed between the first two poles is 150/5 = 30 $m.s^{-1}$ (= u) and between the second two poles is 150/3 = 50 $m.s^{-1}$ (= v)

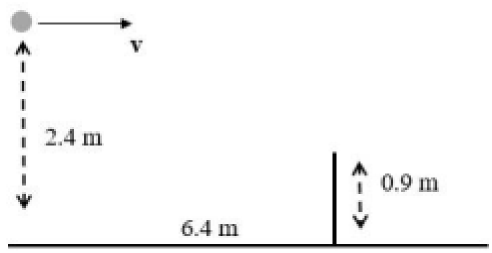
The time taken to make this change is 2 ½ + 1 ½ seconds = 4 s.

Then using v = u + a.t we have that $a = \left(\frac{v - u}{t}\right) = \left(\frac{50 - 30}{4}\right) = 5\ m.s^{-2}$

6.13 Problem

A tennis player, serves a ball horizontally from a height of 2.4 m. The distance from the service line to the net is 6.4 m, and the net, in the middle is 0.9 m high. What is the minimum speed that the ball must be serves at to clear the net, in $m.s^{-1}$? You can neglect air resistance.

Solution B



First thing is to draw a diagram and transfer all the data to it.

The horizontal speed does not change, so what's needed is the time of flight, then knowing the distance one can find the speed.

Vertically: initial speed = 0,
Distance fallen = 2.4 – 0.9 m = 1.5 m
Acceleration = 10 $m.s^{-2}$ then using $x = ut + ½at^2$ we get that $3 = 10t^2$
So t = 0.55 secs. Then since v = x/t = 6.4/0.55 = 11.7 $m.s^{-1}$

There are several other ways to solve this, including using energy. The problem also appears in a variety of different guises, see for example Problem 6.4.

6.14 Problem

Peter runs two laps of a circuit. The first he runs slowly as a 'warm up' lap at an average speed U. At what speed V must he run the second lap, so that his average speed for the two laps is $2U$?

A $V = U$ B $V = 2U$ C $V = 3U$ D $V = 4U$
E He cannot run fast enough to get an average speed for the two laps of $2U$

Solution E

This is a very difficult problem, with an unexpected answer! The most common mistake is to try and average the average speeds. So for example:

If Peter's first lap's average speed is U, then one can't say that his average for two laps is

$2U \neq \frac{3U + U}{2}$ Since average speed is total distance/total time

So if he runs the first lap at $U = \frac{D}{t}$ then in order to run two laps at an average of $2U$ means that for him to do this he will need to cover two laps, a distance of $2D$ in time of t. This of course means that he has no time left to run the second lap! Even at the speed of light, his average speed < $2u$.

6.15 Problem

The displacement time graph for two particles A and B are straight lines inclined at angles of 30° and 60^0 with the time axis. The ratio of velocities $V_A{:}V_B$ is:

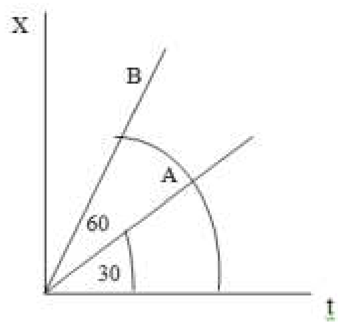
A 1:2 B 1:√3 C √3:1 D 1:3 E 3:1

Solution D

$$V_B = \frac{\delta X_B}{\delta t} = \tan 60 = \sqrt{3}$$

$$V_A = \frac{\delta X_A}{\delta t} = \tan 30 = \frac{1}{\sqrt{3}}$$

$$\text{So } \frac{V_A}{V_B} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$$

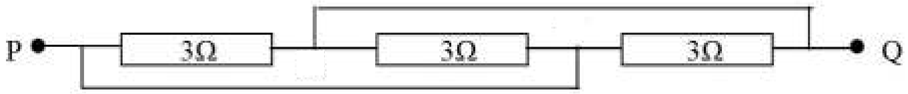


So ratio $V_A{:}V_B = 1{:}3$

7 Electricity

7.1 Problem

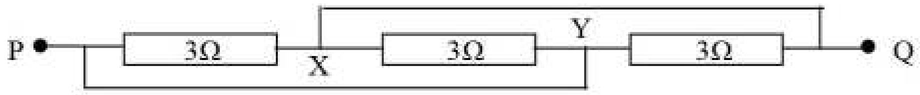
In the circuit shown below the resistance, in ohms (Ω), between the points P and Q is:



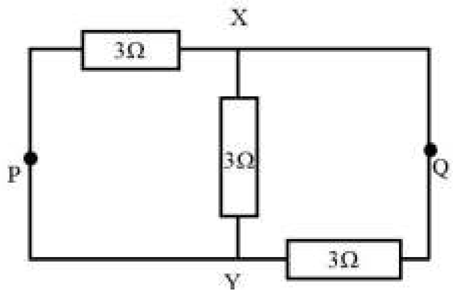
A 3/2 B 9 C 2/3 D 1

Solution D

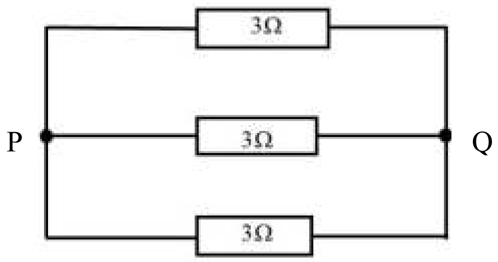
Many have a problem with interpreting this type of diagram correctly. Let's look at the same problem with a few extra labels, X and Y:



Looking at the upper wire, the RHS has X connected to Q, which means that Q and X are the same point! The same applies to the lower wire, Y and P are the same point. So we could redraw the diagram as follows:

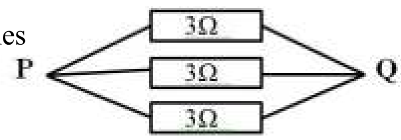


Which is another way of asking the same question, but it becomes easier to see the obvious. By simply sliding Y to P and X to Q you get:



So the equivalent circuit is simply three, 3Ω resistors in parallel, so the equivalent resistance is 1Ω.

Students have got used to drawing circuits, as shown above, with straight lines at right angles to each other. But are many other ways of doing this, especially if a student is drawing a free hand sketch!



Discuss the difference between these diagrams, especially the concept of a "line" being equivalent to a "point". Try drawing a few freehand sketches of different, but equivalent circuits.

7.2 Problem

N identical cells of emf ***E*** and internal resistance *r*, are connected in parallel. This combination is then connected to an external resistance *R*. The current in *R* is:

A $\dfrac{\mathcal{E}}{R+r}$ B $\dfrac{\mathcal{E}}{R+Nr}$ C $\dfrac{\mathcal{E}}{R+\frac{r}{N}}$ $\dfrac{\mathcal{E}}{R+\frac{N}{r}}$

Solution C

For 2 resistors in parallel we have $\frac{1}{r_i} = \frac{1}{r} + \frac{1}{r} = \frac{2}{r} \Rightarrow r_i = \frac{r}{2}$. Similarly if 3 resistors are connected in parallel, r_i becomes $r_i = \frac{r}{3}$ clearly for N resistors in parallel the effective resistance r_i will be $r_i = \frac{r}{N}$

The equation for the circuit is: $E = i(R + r_i)$ so $i = \frac{E}{\left(R + \frac{r}{N}\right)}$

7.3 Problem

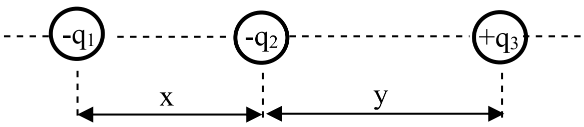
The diagram (not drawn to scale) below shows three small metallic balls carrying charges of $-q_1$, $-q_2$ and $+q_3$. They are on an insulated stands in the same straight line. The magnitude of the net electrostatic force experienced by charge q_2 due to the presence of the other two charges can be expressed as:

A $kq_2(\frac{q_1 - q_3}{x^2y^2})$

B $kq_2(\frac{q_1 + q_3}{x^2y^2})$

C $kq_2(\frac{q_1y^2 - q_3x^2}{x^2y^2})$

D $kq_2(\frac{q_1y^2 + q_3x^2}{x^2y^2})$



Solution D

Let the force between q_1 and q_2 be F_X, and that between q_2 and q_3 F_Y. Inspection shows that F_X pushes q_2 to the right (repulsive force), and F_Y pulls F_Y to the right (attractive force). This means that the resultant force F_R on q_2 is $F_R = F_X + F_Y$

$$F_X = \frac{kq_1q_2}{x^2} \text{ and } F_Y = \frac{kq_2q_3}{y^2} \text{ then } F_R = \frac{kq_1q_2}{x^2} + \frac{kq_2q_3}{y^2} = kq_2\left(\frac{q_1y^2 + q_2x^2}{x^2y^2}\right)$$

7.4 Problem

A battery of emf 12V has an internal resistance of 3 Ω. Three of these are connect in parallel to an external resistor of 5 Ω. What is the power dissipated in the 5 Ω resistor?

A 24 W B 11.25 W C 20 W D 8.89 W

Solution C

Three cells in parallel means that the Total $E_T = 12V$ and the resistance the cells R_C can be found:

$1/R_C = ⅓ + ⅓ + ⅓$ means that $R_c = 1Ω$.

So total resistance $R_T = 1\Omega + 5\Omega = 6\Omega$. This means that the current in the circuit is 2A.

Power = $I^2R = 2^2 \times 5 = 20W$

7.5 Problem

S and T are two charged spheres placed on insulated stands. The charges on S and T are – 2 μC and + 6 μC respectively. The spheres are allowed to touch each other and then returned to their original positions.

Which of the following statements is true.

A S has gained 2×10^6 electrons   B T has gained 2×10^6 electrons
C S has lost 2.5×10^{13} electrons   D T has lost 2.5×10^{13} electrons

Solution C

The charge on a single electron is $e = 1.6 \times 10^{-19}$ C

1 C = 6.25×10^{18} electrons (1 C = 1/e electrons) so 1μC = 6.25×10^{12} electrons

When the conductors are touched electrons flow from S to T so that when they are separated again, each carries a charge of 2μC. This means that 4μC of electrons flowed from S to T,

$4 \times 6.25 \times 10^{12} = 2.5 \times 10^{13}$ electrons.

7.6 Problem

The RMS value of alternating current which produces heat in a given resistor at twice the rate as a direct current of 3A is in amperes:

A 1.5   B $\sqrt{6}$ C $2\sqrt{3}$ D $3\sqrt{2}$

Solution D

The RMS current = to the equivalent DC current.

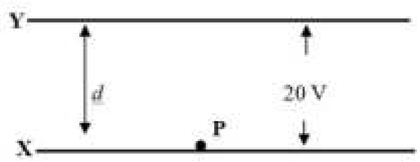
Energy generated $Q = I^2R$, so for 3A, Q = 9R. So for twice the energy, $2Q = i^2R$

ie $18R = i^2R$. So $i^2 = 18$ and $i = \sqrt{18} = \sqrt{(2 \times 9)} = 3\sqrt{2}$.

7.7 Problem

Questions **7.71** and **7.72** refer to the diagram below. It shows two parallel plates a distance *d* apart a potential difference of 20V between them with Y at the higher potential. A small particle P of mass 6×10^{-12} kg carrying a charge of –9 μC is released from plate X. Neglect gravitational effects.

7.71 The speed *u* with which P reaches plate Y is:



A 7 746 $m.s^{-1}$
B 1 225 $m.s^{-1}$
C 30 $m.s^{-1}$
D unable to calculate it unless *d* is known,

Solution 7.71 **A**

PD, in volts, is equal to the work per unit charge,

so: $Vq = Energy = \frac{1}{2} mu^2$

to get: $u^2 = \frac{2Vq}{m} = \frac{2\times 20\times 9\times 10^{-6}}{6\times 10^{-12}} = 60\times 10^6 \Rightarrow u = 7746 m.s^{-1}$

7.72 If the speed with which it reached plate Y above was u, and the distance between the plates is now increased to $2d$, the speed with which it reaches plate Y after being released from X is now:

A $2u$ B u C $u/2$ D $\sqrt{2u}$

Solution 7.72 **B**

As is shown in the calculation above, the speed is independent of the distance between the plates, so the speed remains at u

7.8 Problem

A charge Q is divided in to two parts q and $(Q-q)$. What will the ratio q/Q for the force between them to become the maximum?

A 1/8 B ½ C ¼ D none of the above

Solution **B**

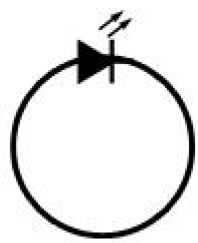
23. (C)

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q(Q-q)}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{qQ-q^2}{r^2}$$

The force will be maximum if, $\frac{dF}{dq} = 0 \quad \frac{1}{4\pi\varepsilon_0}\frac{Q-2q}{r^2} = 0$

$$\frac{1}{4\pi\varepsilon_0}\frac{Q-2q}{r^2} = 0 \quad \frac{1}{4\pi\varepsilon_0}\frac{Q-2q}{r^2} = 0 \quad \text{so } Q-2q = 0 \Rightarrow Q = 2q \Rightarrow \frac{q}{Q} = \frac{1}{2}$$

Problem 7.9



A light emitting diode, LED, is connected to a horizontal coil as shown on the right. A bar magnet is moved within the coil to generate an electric current. Which one of the following actions will make the LED light up? The bar magnet is:

A dropped vertically, N-pole down, through the plane of the coil
B dropped vertically, S-pole down, through the plane of the coil
C rotated horizontally in a clockwise direction in the plane of the coil,
D rotated horizontally in an anti-clockwise direction in the plane of the coil

Solution 7.71 A

PD, in volts, is equal to the work per unit charge,

so: $Vq = Energy = \frac{1}{2} mu^2$

to get: $u^2 = \frac{2Vq}{m} = \frac{2 \times 20 \times 9 \times 10^{-6}}{6 \times 10^{-12}} = 60 \times 10^6 \Rightarrow u = 7746 m.s^{-1}$

7.72 If the speed with which it reached plate Y above was *u*, and the distance between the plates is now increased to 2*d*, the speed with which it reaches plate Y after being released from X is now:

A 2 *u* B *u* C *u*/2 D $\sqrt{2u}$

Solution 7.72 B

As is shown in the calculation above, the speed is independent of the distance between the plates, so the speed remains at u

7.8 Problem

A charge Q is divided in to two parts *q* and (*Q-q*). What will the ratio q/Q for the force between them to become the maximum?

A 1/8 B ½ C ¼ D none of the above

Solution B

23. (C)

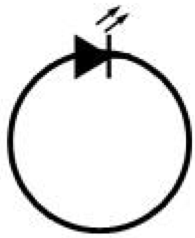
$$F = \frac{1}{4\pi\varepsilon_0} \frac{q(Q-q)}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{qQ - q^2}{r^2}$$

The force will be maximum if, $\frac{dF}{dq} = 0 \quad \frac{1}{4\pi\varepsilon_0} \frac{Q-2q}{r^2} = 0$

$$\frac{1}{4\pi\varepsilon_0} \frac{Q-2q}{r^2} = 0 \quad \frac{1}{4\pi\varepsilon_0} \frac{Q-2q}{r^2} = 0 \quad \text{so } Q - 2q = 0 \Rightarrow Q = 2q \Rightarrow \frac{q}{Q} = \frac{1}{2}$$

Problem 7.9

A light emitting diode, LED, is connected to a horizontal coil as shown on the right. A bar magnet is moved within the coil to generate an electric current. Which one of the following actions will make the LED light up? The bar magnet is:



A dropped vertically, N-pole down, through the plane of the coil
B dropped vertically, S-pole down, through the plane of the coil
C rotated horizontally in a clockwise direction in the plane of the coil,
D rotated horizontally in an anti-clockwise direction in the plane of the coil

8 Modern Physics

8.1 Problem

The Large Hadron Collider (LHC) at CERN is designed to accelerate groups of protons around a large circular ring. At any moment, there will be 3 000 groups in the ring and each group will contain about 10^{11} protons. All the protons go around the ring 10^4 times per second. What is the best estimate of the current in the ring?

A 50 µA B 160 µA C 500 mA D 1.6 A E 160 A

Solution C

Current is rate flow of charge: q/t. Total charge = $3000 \times 10^{11} \times 1.6 \times 10^{-19} = 4.8 \times 10^{-5}$ C

Frequency 10^4 Hz; ie $t = 10^{-4}$ sec. this means that the current is 0.48A = 480mA – close enough – mainly because you never know exactly how many protons there are etc.

8.2 Problem

The Sun's power output is 3.8×10^{26} W. Approximately 10% of this is given out in the form of visible light. Take the average wavelength of visible light to be 500 nm. What is the approximate number of photons of visible light that are given out per day by the Sun?

A 10^{44} B 10^{45} C 10^{49} D 10^{50} E 10^{53}

Solution C

A single photon has energy $E = hf = hc/\lambda = 6.63 \times 10^{-34} \times 3 \times 10^8 / 500 \times 10^{-9} = 3.98 \times 10^{-19}$ J

10% of solar power = 3.8×10^{25} W. So $3.8 \times 10^{25} / 3.98 \times 10^{-19} = 9.56 \times 10^{43}$ photons/sec. (10^{44})

So in a day $3\,600 \times 24 \times 9.56 \times 10^{43} = 8.25 \times 10^{48}$ photons; say 10^{49} photons per day

8.3 Problem

A proton and an alpha particle are accelerated through the same potential difference. The ratios of their de-Broglie wavelengths ($\lambda p/\lambda\alpha$) will be:

A 1 B $\frac{2}{\sqrt{2}}$ C $2\sqrt{2}$ D 2 E $\frac{1}{2}$

Solution D

de Broglie wavelength; $\lambda = \frac{h}{P}$ As both the alpha particle and the proton have been accelerated through the same PD, they will have the same energy; $E = \frac{P^2}{2m}$

So for the alpha particle and for the proton we have $P_P^2 = 2m_pE$

$$\text{So } P_\alpha^2 = 2m_\alpha E \quad \frac{P_\alpha^2}{P_P^2} = \frac{2m_\alpha}{2m_P} = \frac{2\times 4}{2\times 1} = 4$$

$$\sqrt{\frac{P_\alpha^2}{P_P^2}} = \frac{P_\alpha}{P_P} = 2 \text{ so } P_\alpha = 2P_P$$

$$\text{Now } \lambda_\alpha = \frac{h}{P_\alpha} = \frac{h}{2P_P} \text{ and } \lambda_P = \frac{h}{P_P}$$

$$\text{so } \frac{\lambda_P}{\lambda_\alpha} = 2$$

8.4 Problem

Light of two different frequencies whose photons have energies of 1.0 eV and 2.5 eV respectively, illuminate of metal whose work function is 0.5 eV. The ratio of the maximum speed of the emitted electrons will be:

A 1: $\sqrt{2}$ B 1:2 C 1:4 D 1:8

Solution B

The kinetic energy of photo electrons is given by:

$$E_K = E - W_F$$

where E is the energy of the incident photon and W_F is the work function of the metal.

For the 2.5eV photon: ½ $mv^2 = 2.5 - 0.5 = 2$ eV

For the 1 eV photon: ½ $mu^2 = 1 - 0.5 = 0.5$eV

Then $\frac{u^2}{v^2} = \frac{0.5}{2} = \frac{1}{4}$ so $\frac{u}{v} = \frac{1}{2}$

8.5 Problem

Which one of the following expressions correctly gives energy E of a photon of wavelength λ and frequency f? (Planck's constant = h)

A $E = hcf$ B $E = \frac{f\lambda}{h}$ C $E = \frac{h\lambda}{c}$ D $E = \frac{hc}{\lambda}$

Solution D

$E = hf$. From the wave equation we have $c = f\lambda$ so $f = \frac{c}{\lambda}$ and so $E = \frac{hc}{\lambda}$

8.6 Problem

What is the maximum kinetic energy (in eV) of a photoelectron emitted from a surface whose work function is 5 eV when illuminated by a light whose wavelength is 200 nm?

A 1.90 eV B 1.21 eV C 3.10 eV D zero

Solution B

The kinetic energy, E_K, of the photoelectron is given by the equation:

$E_K = E_{PHOTON}$ –Work function, φ, or

$E_K = hf - \varphi$. From the wave equation $c = f\lambda$, so

$$f = \frac{v}{\lambda} = \frac{3.0\times10^{8}\,\text{ms}^{-1}}{200\times10^{-9}\,\text{m}} = 1.5\times10^{15}\,\text{Hz}$$

ie. $E_K = hf - \varphi$

$$= \frac{(6.63\times10^{-34}\,\text{Js})(1.5\times10^{-15}\,\text{Hz})}{1.6\times10^{-19}\,\text{J eV}^{-1}} - 5\text{eV}$$

$$= 6.21\text{eV} - 5\text{eV} = 1.21\text{eV}$$

8.7 Problem

The ratio of momenta of an electron and an α-particle which are accelerated from rest by a potential difference of 100 V is:

A $\sqrt{\left(\frac{m_e}{2m_\alpha}\right)}$ B $\sqrt{\left(\frac{m_e}{m_\alpha}\right)}$ C $\sqrt{\left(\frac{2m_e}{m_\alpha}\right)}$ D 1

Solution B

As both the electron and the alpha particle are accelerated through the same PD, they have the same energy:

$$\tfrac{1}{2}m_\alpha v_\alpha^2 = \tfrac{1}{2}m_e v_e^2 \Rightarrow \frac{v_e}{v_\alpha} = \sqrt{\frac{m_\alpha}{m_e}}$$

$$\textit{but momentum} = mv, \quad \therefore \quad \frac{m_e v_e}{m_\alpha v_\alpha} = \frac{m_e}{m_\alpha}\sqrt{\frac{m_\alpha}{m_e}} = \sqrt{\frac{m_e}{m_\alpha}}$$

8.8 Problem

What is the electric field strength required (in V. m^{-1}) to just hold a water droplet with a diameter of 1 x 10^{-6} m if it is carrying a charge of one electron, 1.6 x 10^{-19} C ?

A 3.27 B 32.7 C 32.7 x 10^{3} D 3.27 x 10^{4}

Solution D

Diameter of drop = 1×10^{-6} m, so radius = 5×10^{-7} m. Vol = $4/3\pi r^3 = 5.24 \times 10^{-19}$ m^3

Now 1 m^3 = 1000 *l* and 1 *l* water has mass of 1 kg, so 1 m^3 has a mass of 10^3 kg and weighs 10^4 N

Therefore, the weight of the droplet is = $5.24 \times 10^{-19} \times 10^4$ N = 5.24×10^{-15} N

Electrostatic force on the electron: F = e x E, so weight W = eE

ie. $E = W/e = 5.24 \times 10^{-18}$ N/1.6×10^{-19} so $E = 3.27 \times 10^4$ V.m^{-1}

Problem

The age of wood can be found by comparing the amount of carbon-14 a sample contains to the amount of carbon-14 in a fresh piece of wood. Such a piece of wood contains 8 times the amount of carbon-14 as a sample from an ancient campfire. How many years ago was the campfire burning if the half-life of carbon-14 is 5 600 years?

A 44 800 B 22 400 C 16 800 D 11 200

Solution C

At time T = 0 was when the wood burnt.
After 5 600 years ½ the C_{14} is left
After 11 200 years ¼ the C_{14} is left, and
After 16 800 years $^1/_8$ the C_{14} is left.

This works very nicely, but when the exact time is not an integer multiple of the half-life a different approach is needed. The change δN in the number of nuclei N present occurring in a short time δt is proportional to both N and δt, so

$\delta N = -\lambda N\, \delta t$ where λ is the decay constant. Then dividing by δt we get:

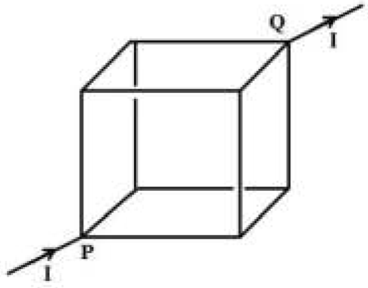
$$\frac{\delta N}{\delta t} = -\lambda N \text{ which has the solution } \frac{N}{N_O} = e^{-\lambda t}$$

This can then be used to solve all these decay problems with the appropriate manipulation.

Fortunately, this is not in the CAPS curriculum!!

9 Additional Examples covering all topics. These are more difficult!

9.1 Problem



The diagram below shows a cubic network of 12 of resistance wires, each with a resistance of R Ω. A current I passes through the network from P to Q. What is the effective resistance between P and Q?

A 12R Ω B $\frac{5R}{12}$ Ω C $\frac{7R}{6}$ Ω

D $\frac{5R}{6}$ Ω E $\frac{R}{12}$ Ω

Solution **D**

It is clear that to draw the equivalent circuit is extremely difficult, and then not easy to use either.

It is also clear that the PD between P and Q can be assumed to be V volts.

Then there are 3 paths from P and each splits in two, these splits rejoin and so there are 3 paths that meet at Q .

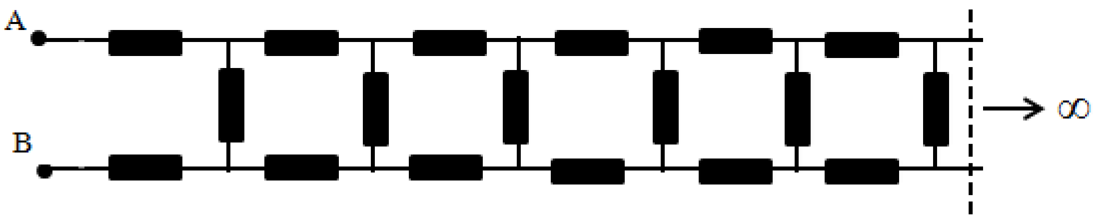
If the current I is assumed to be 6 A, (or 12 A) then the current breaks in to 2 A each at P and 1 A at the split after that

So therefore the PD across one "arm" of the cube will be, 2R, R and 2R since I = 2, 1 and 2 A respectively and using V = IR gives V = 2R + R + 2R = 5R then effective resistance $R_{PQ} = V/I = 5R/6$

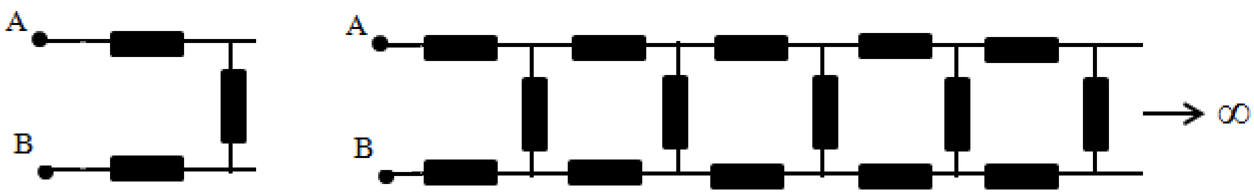
9.2 Problem

Find the effective resistance, resistance between A and B of an infinitely ladder of resistors, each of resistance r Ω.

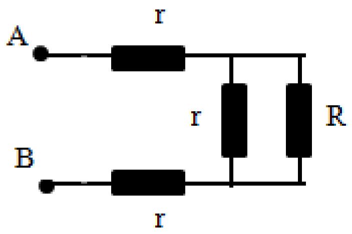
Solution  = r Ω



The above diagram shows an infinite "ladder" of resistors of value r Ω each. If we were to cut at the broken line we would get two pieces as shown: piece **R** to the right and piece **L** to the left

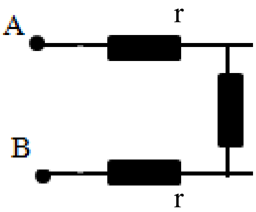


Adding or removing such a part has no effect on the resistance of the infinite ladder. So assume that the resistance between points A and B is R Ω, then we only need to solve the following circuit:



Considering the parallel arrangement and assuming that R_P is the resistance of the parallel combination, we get

$\frac{1}{R_P} = \frac{1}{r} + \frac{1}{R}$ so $R_P = \frac{rR}{r+R}$. We can now redraw the circuit as:



$\frac{rR}{r+R}$ Then assuming $R_{AB} = R$ as before we get that $R = 2r + \frac{rR}{r+R}$

This reduces to $R^2 = 2r^2 + 2rR$

Or $R^2 - 2rR - 2R^2 = 0$. This quadratic can be solved using:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2} \text{ to get } R = \frac{2r \pm \sqrt{4r^2 + 8r^2}}{2} = \frac{2r \pm \sqrt{12r^2}}{2} = \frac{2r \pm 2r\sqrt{3}}{2} = r(1 + \sqrt{3})$$

So the resistance of the infinite ladder of r Ω resistors: $R = r(1 + \sqrt{3})$.

9.3 Problem

A proton of mass m and charge +e is moving in a circular orbit in a magnetic field with energy 1MeV. What would be the energy of an α-particle (mass 4m and charge +2e) so that it can revolve in the path of the same radius

A 0.5 MeV B 1 MeV C 2 MeV D 4 MeV.

Solution B

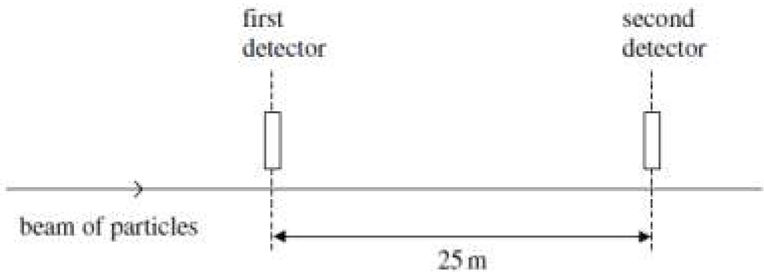
Now for circular motion in a magnetic field $F = qvB = \frac{mv^2}{r} = qB = \frac{mv}{r} = \frac{p}{r} = \frac{\sqrt{2mE}}{r}$

then $r = \frac{\sqrt{2mE}}{qB}$ and $r^2 = \frac{2mE}{q^2B^2}$ so $E = \frac{(rqB)^2}{2m}$ then since q and B are the same

$E \propto \frac{q^2}{2m}$ doubling the charge and making the mass 4x larger leaves the energy the same.

9.4 Problem

A beam of identical particles moving at speed of 0.98c is directed along a straight line between two detectors 25 m apart.



The particles are unstable and the intensity of the beam at the second detector is a quarter of the intensity at the first detector. The half-life of the particles is:

A 17×10^{-8} s B 1.7×10^{-8} s C 8.5×10^{-9} s D 4.25×10^{-9} s

Solution C

Distance between detectors in rest frame of particles $= 25\left(1 - 0.98^2\right)^{\frac{1}{2}} = 5.0m$

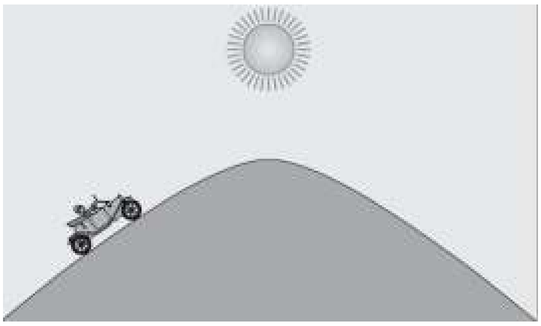
Time taken in rest frame of particles $= \frac{distance}{speed} = \frac{5}{0.98c} = 1.7x10^{-8} s$

Time taken for the intensity to decrease to a ¼ = two half-lives $= \frac{1.7x10^{-8}}{2} = 8.5x10^{-9} s$

ie. One half-life = 8.5×10^{-9} s

9.5 Wertheimer's Problem

Wertheimer wrote Einstein a letter with the following problem:



An old clattery auto is to drive a stretch of 2 km, up and down a hill. Because it is so old, it cannot drive the first km — the ascent —faster than with an average speed of 15 km per hour. Question: How fast does it have to drive the second km — on going down, it can, of course, go faster—in order to obtain an average speed (for the whole distance) of 30 km per hour?

Solution

Einstein fell for this teaser

Wertheimer's thought problem suggests the answer might be 45 or even 60 km per hour. But that is not the case. Even if the car broke the sound barrier on the way down, it would not achieve an average speed of 30 km an hour. Don't be worried if you were fooled, Einstein was at first too. Replying "Not until calculating did I notice that there is no time left for the way down!"

How long does it take the old car to reach the top of the hill? The road up is one long. The car travels 15 km per hour, so it takes four minutes (one hour divided by fifteen) to reach the top. How long does it take the car to drive up and down the hill, with an average speed of 30 kms per hour? The road up and down is two kms long. Thirty kms per hour translates into two kms per four minutes. Thus, the car needs four minutes to drive the entire distance. But these four minutes were already used up by the time the car reached the top.

See also Problem 6.7

10 Data Sheet

10.1 Physical Constants

Quantity	Symbol	Value
Acceleration due to gravity*	g	$10\ m.s^{-2}$
Universal Constant of Gravitation	G	$6.67 \times 10^{-11}\ N.m^2.kg^{-2}$
Speed of light in a vacuum	c	$3.0 \times 10^8\ m.s^{-1}$
Planck's Constant	h	$6.63 \times 10^{-34}\ J.s$
Coulomb's Constant	k	$9.0 \times 10^9\ N.m^2\ C^{-2}$
Charge on Electron	e	$-1.6 \times 10^{-19}\ C$
Electron mass	m_e	$9.11 \times 10^{-31}\ kg$

* For simplicity use this unless otherwise stated

10.2 Other

Earth – diameter = 12 800 km
Sun – diameter = 1.4×10^9 m
1AU = 1.5×10^{11} m

10.3 Mechanics

10.3.1 Motion

$v_f = v_i + a\Delta t$ $v_f^2 = v_i^2 + 2a\Delta x$ $\Delta x = v_i\Delta t + \frac{1}{2} a\Delta t^2$ $\Delta x = \left(\frac{v_f + v_i}{2}\right)\Delta t$

10.3.2 Force

$F_{RES} = ma$ $p = mv$ $F\Delta t = \Delta p$ $f_k = \mu_k N$ $F = \frac{mv^2}{r}$

10.4.3 Work, Energy, Power

$W = F\Delta x\cos\theta$ $E_p = mgh$ $E_k = \frac{1}{2} mv^2$ $W = \Delta E_p + \Delta E_k$

Power = $W/\Delta t$ Power = Fv

10.4 Sound, Waves and Light

$v = f\lambda$ $T = 1/f$ $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ $f = \frac{c \pm v}{c \quad v_0} f_0$

Speed of sound in air = $340\ m.s^{-1}$

Frequency (in Hz):
IR from $\sim 5 \times 10^{11} - 4 \times 10^{14}$
Visible from $\sim 4 \times 10^{14} - 8 \times 10^{14}$
UV from $\sim 8 \times 10^{14} - 5 \times 10^{15}$
X-rays $\sim 5 \times 10^{15} - 3 \times 10^{21}$

10. 5 Heat

$\Delta Q = mc\Delta\theta$ Specific Heat Capacity $c = \frac{\Delta Q}{m\Delta\theta}$ $J.kg^{-1}.K^{-1}$

10.6 Electricity

10.6.1 Electrostatics

$F = \frac{kQq}{r^2}$ $E = \frac{kQ}{r^2}$ $E = \frac{F}{q}$ $E = \frac{V}{d}$

10.6.2 Circuits

$V = IR$ $\mathcal{E} = I(R + r)$ $q = I\Delta t$ $C = \frac{Q}{V}$

$R = r_1 + r_2 + r_3 + \ldots$ $\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \ldots$

Power $= I^2R = V^2/R = VI$

Work $W = VQ = VI\Delta t$

11 Gravitation

$F = \frac{GMm}{r^2}$

12 Modern Physics

De Broglie wavelength $\lambda = \frac{h}{P}$ and $E = hf$

Photoelectric effect $E_K = hf - \varphi$

Wave equation: $c = f\lambda$

11 SI Units

11.1 Base units

Base Quantity	**Name**	**Symbol**
Length	Metre	m
Mass	Kiliogram	kg
Time	Second	s
Electric Current	Ampere	A
Temperature	Kelvin	K
Quantity of Substance	Mole	mol
Luminous Intensity	Candela	cd

11.2 Derived Units

Quantity	Name	Symbol	Expression in terms of Base Units	ITO other SI Units
Acceleration		a	$m.s^{-2}$	
Plane angle	Radian	rad	m/m	
Frequency	Hertz	Hz	s^{-1}	
Force	Newton	N	$kg.m.s^{-2}$	J/m
Pressure	Pascal	Pa	$kg.m.s^{-2}$	N/m^2
Energy: work	Joule	J	$kg.m^2. s^{-2}$	N.m
Power	Watt	W	$kg.m^{2.} s^{-3}$	J/m
Electric charge	Coulomb	Q	A.s	A.s
Electric Potentail (emf)	Volt	V	$\frac{kg.m^2}{A.s^3}$	W/A
Capacitance	Farad	F	$\frac{A^2.s^4}{kg.m^2}$	C/V
Electrical Resistance	Ohm	Ω	$\frac{kg.m^2}{A^2.s^3}$	V/A
Magnetic flux	Weber	Wb	$\frac{kg.m^2}{A^2.s^2}$	V.s
Magnetic field intensity	Tesla	T	$\frac{kg}{A.s^2}$	Wb/m^2
Inductance	Henry	H (or L)	$\frac{kg.m^2}{A^2.s^3}$	Wb/A

12 Contributors

11.1 National Science and Technology Forum; proSET
11.2 Department of Science and Technology, Rep. of South Africa
11.3 Institute of Physics, IOP for Africa Project, United Kingdom
11.4 British High Commission to South Africa
11.5 The South African Institute of Physics, SAIP.

13 Answers for MCQs - Volume 1

#3 Mechanics

1 C	2 B	3 Y	4 C	5 C	6 C	7 D	8 C	9 C	10 W
11 C	12 C	13 D	14 D	15 C	16 B	17 B	18 C	19 A	20 B
21 D	22 D	23 D	24 A	25 D	26 D	27 B	28 A	29 D	30 B
31 C	32 D	33 C	34 B	35 A	36 C	37 D	38 C	39 C	40 D
41 B	42 D	43 B	44 B	45 A	46 B	47 D	48 E	49 D	50 C
51 B	52 B	53 C	54 A	55 D	56 D	57 C	58 D	59 E	60 B
61 C	62 D	63 A	64 C	65 C	66 C	67 B	68 C	69 B	70 B
71 C	72 C	73 W	74 B	75 B	76 D	77 B	78 B	79 D	80 B
81 B	82 D	83 E	84 D	85 D	86 A	87 B	88 C	89 B	90 D
91 C	92 C	93 C	94 A	95 C	96 B	97 A	98 D	99 D	100 E
101 C	102 D	103 D	104 D	105 C	106 E	107 B			

#4 Electricity

1 C	2 C	3 B	4 C	5 C	6 A	7 B	8 D	9 A	10 B
11 D	12 C	13 B	14 C	15 C	16 C	17 D	18 A	19 B	20 D
21 C	22 A	23 C	24 C	25 C	26 D	27 B	28 A	29 A	30 A
31 C	32 A	33 B	34 B	35 C	36 B	37 A	38 A	39 C	40 B
41 E	42 A	43 C	44 C	45 D	46 D	47 B	48 E	49 C	50 C
51 B	52 A	53 B							

#5 Additional Mechanics

1 D	2 A	3 B	4 B	5 B	6 C	7 A	8 A	9 X	10 D
11 A	12 C	13 A	14 A						

#6 Additional Electricity

1 C	2 B	3 C	4 A	5 B	6 A	7 D	8 C	9 A	10 B

Notes

5.9 No answers given
6.9 Answer should be 112.5 – the decimal point has been incorrectly placed.

13 Answers for MCQs - Volume 2

#3 General Properties of Matter

1 C	2 A	3 C	4 C	5 B	6 B	7 C	8 C	9 E	10 A
11 C	12 D	13 D	14 D	15 B	16 A	17 A	18 C	19 B	20 B
21 D	22 A	23 C	24 C	25 D	26 D	27 A	28 B	29 C	30 C
31 C	32 B								

#4 Waves

1 B	2 A	3 C	4 C	5 D	6 A	7 E	8 C	9 A	10 A
11 C	12 D	13 C	14 B	15 C	16 C	17 D	18 A	19 D	20 A
21 B	22 C	23 A	24 C	25 B	26 B	27 E			

#5 Sound

1 C	2 D	3 D	4 C	5 B	6 B	7 C	8 A	9 C	10 B
11 A	12 C	13 C	14 A	15 B					

#6 Light

1 B	2 C	3 D	4 C	5 D	6 C	7 D	8 D	9 A	10 C
11 D	12 C	13 C	14 B	15 D	16 B	17 C	18 C	19 C	20 B
21 A	22 B	23 A	24 A	25 A	26 D	27 C	28 C	29 C	30 D
31 C	32 C	33 D	34 D	35 C	36 D	37 D	38 B	39 B	40 D
41 A	42 D	43 D	44 A	45 B	46 A	47 D	48 A	49 D	50 D
51 C	52 C								

#7Heat

1 B	2 D	3 A	4 D	5 D	6 A	7 A	8 A	9 B	10 C
11 A	12 B	13 D	14 C	15 C	16 B	17 B	18 C	19 A	20 C
21 F									

#8 Modern Physics

1 A	2 D	3 D	4 C	5 B	6 C	7 D	8 C	9 B	10 B
11 D	12 C	13 C	14 C	15 D	16 D	17 E	18 B	19 B	20 A
21 C	22 C	23 ?	24 B	25 C	26 D	27 C	28 *	29 B	30 C
31 D	32 D	33 D							

#9 Additional Problems

1 D	2 C	3 D	4 C	5 D	6 A	7 C	8 B	9 B	10 C
11 B	12 B	13 B	14 **	15	16 B	17 E			

Notes:

* Typo with decimal point – 112.5 is correct

** Given answer A is not correct – need to use K not C° – 1.56×10^5 J.K^{-1}

14 Acknowledgements

14.1 Prof David Wolfe, Emeritus Professor of Physics University of New Mexico and Volunteer Coordinator for South Africa, IOP for Africa Project, UK for providing many problems and lots of support.

14.2 Where the letters DLG appear it is stating that this piece has been adapted from ***An Approach to Promoting Conceptual Understanding of Physics Through Problem-Solving Activities*** by Robert Dufresne, William Leonard and William Gerace, Universality of Massachusetts, Amherst, USA.

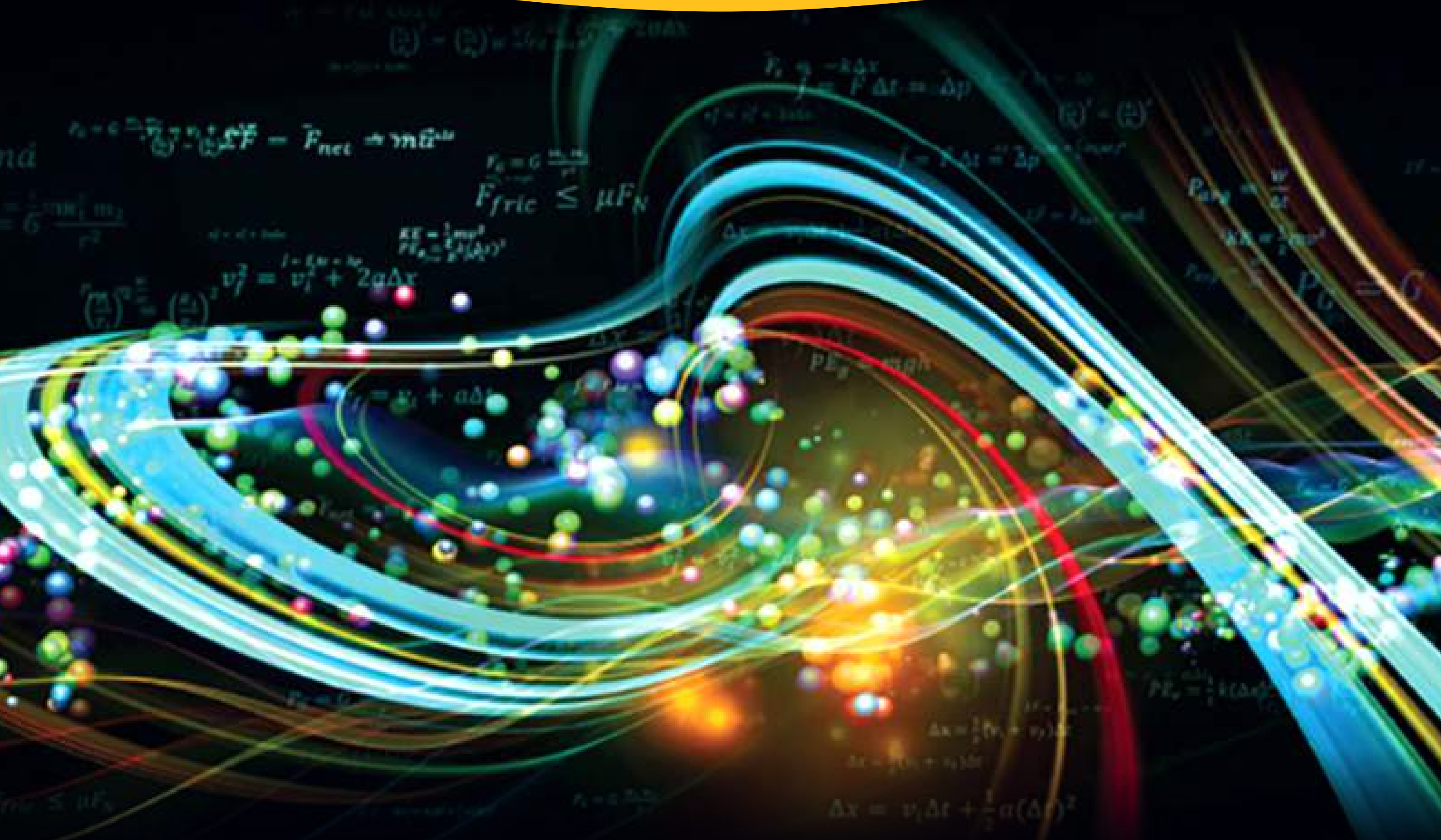
14.3 Dr Sam Ramaila, University of Johannesburg for some of the additional Pedagogical material

14.4 Prof Hartmut Winkler, UJ for providing some problems

14.5 Prof Andre Venter, NMU for providing some problems

14.6 Mr Brian Masara, Executive Officer, South African Institute of Physics for making it possible.

South African Institute of
PHYSICS
THE VOICE OF PHYSICS IN SOUTH AFRICA



PHYSICS PEDAGOGY

PROBLEM SOLVING STRATEGIES USING WORKED EXAMPLES

VOLUME 3 - 2020

Compiled by Case Rijsdijk

CONTACT SOUTH AFRICAN INSTITUTE OF PHYSICS

Postal Address: SAIP, Postnet Suite 165, Private Bag X025, Lynnwood Ridge, 0040, South Africa

Website: http://www.saip.org.za • Email:info@saip.org.za • Phone: 012 841 2655 / 2627

Facebook: https://www.facebook.com/South-African-Institute-of-Physics-1660099704207118/